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ON EQUATIONS OF THREE-DIMENSIONAL LAMINAR BOUNDARY LAYER OF BODIES OF REVOLUTION

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B. M. BULAKH and M. S. SIMKIN
(Leningrad)
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The uniformly accurate equations of a plane uniform laminar boundary layer for a body whose profile is sharply curved, are derived in [1]. In the present paper the results of [1] are generalized for the case of a body of revolution in a supersonic gas flow at incidence. The most important result lies in the fact that parameters of the gas flow in the boundary layer in the domain of sharp curvature of the generatirix of the body of revolution can be defined independently in every meridional plane passing through the axis of symmetry of the body if the curvature radius of the generatrix of the body becomes a quantity of the order of boundary-layer thickness.

1. We consider a certain body of revolution whose curvature $\varkappa$ of the

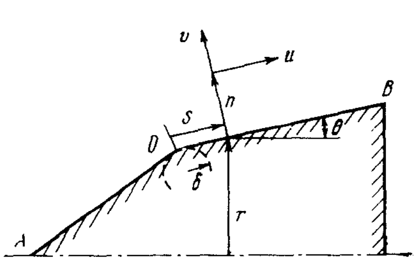

Fig. 1

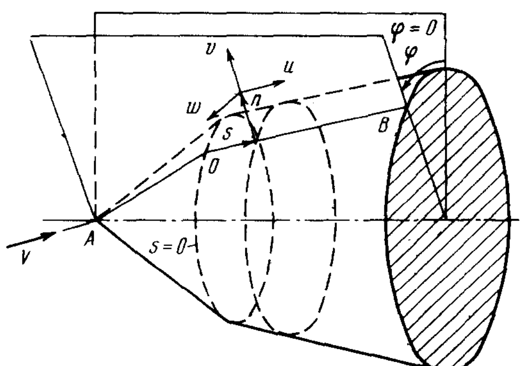

Fig. 2

generatrix AOB (Fig. 1) is a continuous function of the coordinate s, measured along the generatrix from the point O, where $\varkappa$ attains its greatest value $\varkappa_{\max}$, and the radius of curvature, correspondingly, its minimum value $\delta = (\varkappa_{\max})^{-1}$. We take the distance from point A to point O, measured along the the generatrix AOB (Fig. 1), as the characteristic length l_0 and will refer all lengths to l_0.

We shall represent the curvature $\varkappa$ in the vicinity of point O in the form

$$\varkappa = K(S, \delta)\,\delta^{-1}, \qquad S = s\delta^{-1}, \qquad 0 \leqslant K(S, \delta) \leqslant 1 \quad \text{for} \quad -1 \leqslant S \leqslant 1 \tag{1.1}$$

(for details see [1]).

2. Now let a uniform stream of viscous perfect gas flow past a body of rotation having the indicated generatrix. In the system of coordinates in which s is measured along the generatrix of the body, n is normal to it and the angle φ defines the meridional plane (Fig. 2), the equations of continuity, momentum, energy and the equation of state of gas have the form

$$[(r + n\cos\theta)\,\rho u]_s + [(r + n\cos\theta)(1 + \varkappa n)\,\rho v]_n + [(1 + \varkappa n)\,\rho w]_\varphi = 0 \tag{2.1}$$

$$\begin{aligned}
&\varepsilon^{-2}\left[\rho\left(\frac{uv_s}{1+\varkappa n} + vv_n + \frac{wv_\varphi}{r+n\cos\theta} - \frac{\cos\theta\, w^2}{r+n\cos\theta} - \frac{\varkappa u^2}{1+\varkappa n}\right) + p_n\right] = \\
&= \frac{1}{1+\varkappa n}\left\{\mu\left[\frac{1}{1+\varkappa n}(v_s - u\varkappa) + u_n\right]\right\}_s + \left\{(\lambda+2\mu)\, v_n + \lambda\left[\frac{1}{1+\varkappa n}(u_s + \varkappa v) + \right.\right. \\
&\left.\left. + \frac{1}{r+n\cos\theta}(w_\varphi + u\sin\theta + v\cos\theta)\right]\right\}_n + \frac{1}{r+n\cos\theta}\left\{\mu\left[w_n + \right.\right. \\
&\left.\left. + \frac{1}{r+n\cos\theta}(v_\varphi - w\cos\theta)\right]\right\}_\varphi + \mu\left\{\frac{1}{r+n\cos\theta}\left[\sin\theta\left(\frac{1}{1+\varkappa n}(v_s - u\varkappa) + u_n\right) + \right.\right. \\
&\left. + \cos\theta\left(2v_n - \frac{2}{r+n\cos\theta}(w_\varphi + u\sin\theta + v\cos\theta)\right)\right] + \\
&\left. + \frac{2\varkappa}{1+\varkappa n}\left[v_n - \frac{1}{1+\varkappa n}(u_s + \varkappa v)\right]\right\}
\end{aligned} \tag{2.2}$$

$$\begin{aligned}
&\varepsilon^{-2}\left\{\rho\left[\frac{uu_s}{1+\varkappa n} + vu_n + \frac{wu_\varphi}{r+n\cos\theta} + \frac{\varkappa uv}{1+\varkappa n} - \frac{\sin\theta\, w^2}{r+n\cos\theta}\right] + \frac{p_s}{1+\varkappa n}\right\} = \\
&= \frac{1}{1+\varkappa n}\left\{\frac{\lambda+2\mu}{1+\varkappa n}(u_s + \lambda v) + \lambda\left[v_n + \frac{1}{r+n\cos\theta}(w_\varphi + u\sin\theta + v\cos\theta)\right]\right\}_s + \\
&+ \left\{\mu\left[\frac{1}{1+\varkappa n}(v - \varkappa u) + u_n\right\}_n + \frac{1}{r+n\cos\theta}\left\{\mu\left[\frac{1}{r+n\cos\theta}(u_\varphi - w\sin\theta) + \right.\right.\right. \\
&\left.\left. + \frac{w_s}{1+\varkappa n}\right]\right\}_\varphi + \mu\left\{\frac{2\sin\theta}{r+n\cos\theta}\left[\frac{1}{1+\varkappa n}(u_s + \varkappa v) - \frac{1}{r+n\cos\theta}(w_\varphi + \right.\right. \\
&\left.\left. + u\sin\theta + v\cos\theta)\right] + \left(\frac{\cos\theta}{r+n\cos\theta} + \frac{2\varkappa}{1+\varkappa n}\right)\left[\frac{1}{1+\varkappa n}(v_s - u\varkappa) + u_n\right]\right\}
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
&\varepsilon^{-2}\left\{\rho\left[\frac{uw_s}{1+\varkappa n} + vw_n + \frac{w}{r+n\cos\theta}(w_\varphi + u\sin\theta + v\cos\theta)\right] + \frac{p_\varphi}{r+n\cos\theta}\right\} = \\
&= \frac{1}{1+\varkappa n}\left\{\mu\left[\frac{1}{r+n\cos\theta}(u_\varphi - w\sin\theta) + \frac{w_s}{1+\varkappa n}\right]\right\}_s + \left\{\mu\left[w_n + \frac{1}{r+n\cos\theta}(v_\varphi - \right.\right. \\
&\left.\left. - w\cos\theta)\right]_n + \frac{1}{r+n\cos\theta}\left\{\frac{\lambda+2\mu}{r+n\cos\theta}(w_\varphi + u\sin\theta + v\cos\theta) + \lambda\left[v_n + \right.\right. \\
&\left.\left. + \frac{1}{1+\varkappa n}(u_s + \varkappa v)\right]\right\}_\varphi + \mu\left\{\frac{2\sin\theta}{r+n\cos\theta}\left[\frac{1}{r+n\cos\theta}(u_\varphi - w\sin\theta) + \frac{w_s}{1+\varkappa n}\right] + \right. \\
&\left. + \left(\frac{\varkappa}{1+\varkappa n} + \frac{2\cos\theta}{r+n\cos\theta}\right)\left[w_n + \frac{1}{r+n\cos\theta}(v_\varphi - w\cos\theta)\right]\right\}
\end{aligned} \tag{2.4}$$

$$\varepsilon^{-2}\left[\rho\left(\frac{uT_s}{1+\varkappa n}+vT_n+\frac{wT_\varphi}{r+n\cos\theta}\right)-\left(\frac{up_s}{1+\varkappa n}+vp_n+\frac{wp_\varphi}{r+n\cos\theta}\right)\right]=$$
$$=\frac{\sigma^{-1}}{1+\varkappa n}\left(\frac{\mu T_s}{1+\varkappa n}\right)_s+\sigma^{-1}(\mu T_n)_n+\frac{\sigma^{-1}}{(r+n\cos\theta)^2}(\mu T_\varphi)_\varphi+$$
$$+\frac{\sigma^{-1}\sin\theta\mu T_s}{(1+\varkappa n)(r+n\cos\theta)}+\sigma^{-1}\left(\frac{\varkappa}{1+\varkappa n}+\frac{\cos\theta}{r+n\cos\theta}\right)\mu T_n+\Phi \tag{2.5}$$

where
$$\Phi=\mu\left\{\frac{2}{(1+\varkappa n)^2}(u_s+\varkappa v)^2+2(v_n)^2+\frac{2}{(r+n\cos\theta)^2}(w_\varphi+\right.$$
$$+u\sin\theta+v\cos\theta)^2+\left[\frac{1}{1+\varkappa n}(v_s-\varkappa u)+u_n\right]^2+\left[w_n+\right.$$
$$\left.\left.+\frac{1}{r+n\cos\theta}(v_\varphi-w\cos\theta)\right]^2+\left[\frac{1}{1+\varkappa n}w_s+\frac{1}{r+n\cos\theta}(u_\varphi-w\sin\theta)\right]^2\right\}+$$
$$+\lambda\left\{\frac{1}{1+\varkappa n}(u_s+\varkappa v)+v_n+\frac{1}{r+n\cos\theta}(w_\varphi+u\sin\theta+v\cos\theta)\right\}^2 \tag{2.6}$$
$$p=\frac{\gamma-1}{\gamma}\rho T,\ \mu=\mu(T),\ \lambda=\lambda(T),\ \varepsilon=\frac{1}{\sqrt{R}},\quad R=\frac{V_0 l_0\rho_0}{\mu_0}$$

Here u, v, w are the velocity components in the direction of increasing s, n, φ, respectively, ρ is the density, p is the pressure, T is the temperature, σ is the Prandtl number, μ, λ are the coefficients of viscosity, γ is the adiabatic index, R is the Reynolds number formed from the characteristic parameters of flow, r is the distance from the body surface to the axis of symmetry, θ is the angle of inclination of the tangent to the generatrix of the body and the axis of symmetry. Derivatives are denoted by subscripts, for example, $u_n=\partial u/\partial n$.

In equations (2.1)–(2.6) all lengths are referred to l_0, velocities to V_0, density to ρ_0, pressure to $\rho_0 V_0^2$, temperature to $T_0=V_0^2\, c_p^{-1}$ (C_p is the specific heat of the gas at constant pressure), and coefficients of viscosity to μ_0. The notation for the dimensionless quantities is retained also for dimensional ones. As characteristic parameters V_0, ρ_0, μ_0 it is convenient in the present case to take the corresponding values of V, ρ, μ in the stream directly ahead of point O for $\varphi=\pi$ (Fig. 2).

3. As is known, for the solution of the problem of flow past bodies in the case $\varepsilon\ll 1$, one distinguishes the boundary layer – a region of thickness $O(\varepsilon)$ directly adjacent to the body surface and the "external flow" region. In the latter region the solution to the system of Eqs. (2.1)–(2.6) is sought in the form of the asymptotic expansion [2]

$$f(s,n,\varphi,\varepsilon)\sim F_1(s,n,\varphi)+\varepsilon F_2(s,n,\varphi)+\ldots \tag{3.1}$$

Here f stands for u, v, w, p, ρ, T.

In the boundary-layer region, where viscous forces are of the same order as inertial forces, a new variable $N=n\varepsilon^{-1}$ is introduced, and the solution is sought in the form of the asymptotic expansion

$$f(s,n,\varphi,\varepsilon)\sim f_1(s,N,\varphi)+\varepsilon f_2(s,N,\varphi)+\ldots \tag{3.2}$$
$$v\sim\varepsilon v_1+\varepsilon^2 v_2+\ldots,$$

where f stands for u, w, p, ρ, T.

Equations for the first terms of expansion (3.1) are the Euler equations; the equations for the first terms of the expansion (3.2) are the Prandtl equations which have in our case the form

$$(r\rho_1 u_1)_s + (r\rho_1 v_1)_N + (\rho w_1)_\varphi = 0, \; p_{1N} = 0$$

$$\rho_1 [u_1 w_{1s} + v_1 w_{1N} + w_1 r^{-1} (w_{1\varphi} + u_1 \sin\theta)] + r^{-1} p_{1\varphi} = (\mu w_{1N})_N$$

$$\rho_1 (u_1 u_{1s} + v_1 u_{1N} + w_1 r^{-1} u_{1\varphi} - \sin\theta r^{-1} w_1^2) + p_{1s} = (\mu u_{1N})_N$$

$$\rho_1 (u_1 t_{1s} + v_1 t_{1N} + w_1 r^{-1} t_{1\varphi}) - (u_1 p_{1s} + w_1 r^{-1} p_{1\varphi}) =$$

$$= \sigma^{-1} (\mu t_{1N})_N + \mu [(u_{1N})^2 + (w_{1N})^2], \; p_1 = \frac{\gamma - 1}{\gamma} \rho_1 t_1 \tag{3.3}$$

Equations (3.3) are correct to within a factor $1 + O(\varepsilon)$ for $\varepsilon \to 0$ and can be used for the determination of the gas motion in a boundary layer for small but finite ε in those cases when $\varkappa$ does not appreciably exceed unity. However, if $\varkappa \gg 1$, it is appropriate to regard the problem as depending upon two parameters ε and δ [1].

The purpose of what follows is to obtain equations for the motion of the gas in the vicinity of line $s = 0$ (Fig. 2) under the condition $\varepsilon, \delta \to 0$, that are correct to within a factor $1 + O(\varepsilon)$ and suitable for utilization for small but finite ε and δ. When constructing an asymptotic theory of gas motion in the vicinity of line $s = 0$ we shall consider two cases

$$\lim \varepsilon\delta^{-1} = 0, \qquad \varepsilon, \delta \to 0 \text{ and } \lim \varepsilon\delta^{-1} = \beta_0 < \infty, \qquad \varepsilon, \delta \to 0$$

4. The case $\lim \varepsilon\delta^{-1} = 0$ for $\varepsilon, \delta \to 0$. Since in transition from the point of the generatrix, defined by coordinate $S = s\delta^{-1} = -1$ to the points $S = +1$ ($\varphi = \text{const}$), the tangent to the generatrix rotates through a finite angle, the inviscid supersonic flow corresponding to the first term of the expansion (3.1) undergoes a finite change in gas parameters in this segment, in particular of the pressure p. In the boundary layer $p_N = 0$ ahead of and behind certain neighborhood of line $s = 0$, so that also in a layer of thickness $O(\varepsilon)$ in the vicinity of line $s = 0$ the quantity $p = O(1)$ changes by a finite amount; that is

$$p_s = O(\delta^{-1}) \quad \text{or} \quad p_S = O(1), \qquad -1 < S < 1$$

For the other gas parameters we assume that differentiation with respect to $S = s\delta^{-1}$ for $-1 < S < 1$ does not change the order of functions for $\varepsilon \to 0$, $\delta \to 0$. The gas flow in the boundary layer ahead of line $s = 0$ is a flow with a strong shear. Undoubtedly, the flow has the same character also in the vicinity of line $s = 0$. We therefore assume that in the vicinity of line $s = 0$, as well as ahead of it, differentiation of functions with respect to $N = n\varepsilon^{-1}$ does not change the order of functions for $N = O(1)$ and $-1 < S < 1$. Let us transform Eqs. (2.1)–(2.5) to the variables $S = s\,\delta^{-1}$ and $N = n\varepsilon^{-1}$.

We can write the continuity equation (2.1) in the form

$$[(r + \varepsilon N \cos\theta)\rho u]_S + [(r + \varepsilon N \cos\theta)(1 + K\varepsilon\delta^{-1}N)\rho v\delta\varepsilon^{-1}]_N + \delta[(1 + KN\varepsilon\delta^{-1}) \times \rho w]_\varphi = 0 \tag{4.1}$$

Since $\delta\varepsilon^{-1} \to \infty$ as $\delta \to 0$, $\varepsilon \to 0$, $v = O(\varepsilon\delta^{-1})$. Let us introduce $v^* = O(1)$ by the substitution

$$v = \varepsilon\delta^{-1} v^* \tag{4.2}$$

Equation (4.1) takes the form

$$(r\rho u)_S + [r(1 + KN\varepsilon\delta^{-1})\rho v^*]_N + \delta(\rho w)_\varphi = O(\varepsilon) \tag{4.3}$$

Equations (2.2)–(2.5) with (4.2) taken into account, in variables S and N can be presented in the form

$$p_N - \varepsilon\delta^{-1} \frac{K\rho u^2}{1 + KN\varepsilon\delta^{-1}} + (\varepsilon\delta^{-1})^2 \rho \left(\frac{u v_s^*}{1 + KN\varepsilon\delta^{-1}} + v^* v_N^* \right) = O(\varepsilon) \tag{4.4}$$

$$\rho\left[\frac{uu_s}{1+KN\varepsilon\delta^{-1}}+v^*u_N+\varepsilon\delta^{-1}\frac{Kuv^*}{1+KN\varepsilon\delta^{-1}}+\frac{\delta w\,(u_\varphi-\sin\theta w)}{r}\right]+ \qquad \text{(cont.)}$$

$$+\frac{p_s}{1+KN\varepsilon\delta^{-1}}=\delta\,(\mu u_N)_N+O(\varepsilon) \tag{4.5}$$

$$\rho\left(\frac{uw_s}{1+KN\varepsilon\delta^{-1}}+v^*w_N\right)=\delta\left[(\mu w_N)_N-\frac{w\rho\,(w_\varphi+u\sin\theta)}{r}-\frac{p_\varphi}{r}\right]+O(\varepsilon) \tag{4.6}$$

$$\rho\left(\frac{uT_s}{1+KN\varepsilon\delta^{-1}}+v^*T_N\right)-\left(\frac{up_s}{1+KN\varepsilon\delta^{-1}}+p_Nv^*\right)=$$
$$=\delta\,[\sigma^{-1}(\mu T_N)_N+r^{-1}w\,(p_\varphi-\rho T_\varphi)+\mu\,(u_N{}^2+w_N{}^2)]+O(\varepsilon) \tag{4.7}$$

On the surface of the body $N=0$, $u=v^*=w=0$ and Eq. (4.5) is reduced to the form

$$p_S=\delta\,(\mu u_N)_N+O(\varepsilon) \tag{4.8}$$

The left side of (4.8) contains finite quantity, and the right side is infinitely small for $\varepsilon\to 0$, $\delta\to 0$, so that near the wall there is a layer where derivatives of functions with respect to N are of different order than the functions themselves. It follows from (4.8) that in this layer it is necessary to introduce the variable

$$\eta=N\delta^{-1/2}=n\varepsilon^{-1}\,\delta^{-1/2} \tag{4.9}$$

The continuity equation (2.1) shows that $v=O\,(\varepsilon\,\delta^{1/2})$ for $\eta=O\,(1)$. With the substitution $v=\varepsilon\delta^{-1/2}\,v^\circ$ and transition to the variables S, η Eqs. (2.1)–(2.5) take the form

$$(r\rho u)_S+[r\,(1+K\eta\varepsilon\delta^{-1/2})\,\rho v^\circ]_\eta+\delta\,(\rho w)_\varphi=O\,(\varepsilon) \tag{4.10}$$

$$p_\eta-K\,\rho u^2\,\varepsilon\delta^{-1/2}=O\,(\varepsilon\delta^{1/2})+O\,(\varepsilon^2\delta^{-1}) \tag{4.11}$$

$$\rho\left[\frac{uu_S}{1+K\eta\varepsilon\delta^{-1/2}}+v^\circ u_\eta+\frac{Kuv^\circ}{1+K\eta\varepsilon\delta^{-1/2}}\varepsilon\delta^{-1/2}+\delta\,(u_\varphi-w\sin\theta)\,wr^{-1}\right]+$$
$$+\frac{p_S}{1+K\eta\varepsilon\delta^{-1/2}}=(\mu u_\eta)_\eta+\varepsilon\delta^{-1/2}[2K\mu u_\eta-(\mu Ku)_\eta]+O\,(\varepsilon\delta^{-1/2})+O\,(\varepsilon^2\delta^{-1}). \tag{4.12}$$

$$\rho\left(\frac{uw_S}{1+K\eta\varepsilon\delta^{-1/2}}+v^\circ w_\eta\right)=(\mu w_\eta)_\eta-\delta\left[\frac{\rho w\,(w_\varphi+u\sin\theta)}{r}+\frac{p_\varphi}{r}\right]+$$
$$+\mu Kw_\eta\varepsilon\delta^{-1/2}+O\,(\varepsilon\delta^{1/2})+O\,(\varepsilon^2\delta^{-1}) \tag{4.13}$$

$$\rho\left(\frac{uT_S}{1+K\eta\varepsilon\delta^{-1/2}}+v^\circ T_\eta\right)-\left(\frac{up_S}{1+K\eta\varepsilon\delta^{-1/2}}+v^\circ p_\eta\right)=\sigma^{-1}(\mu T_\eta)_\eta+$$
$$+\mu\,(u_\eta{}^2+w_\eta{}^2)+\delta\,[r^{-1}w\,(p_\varphi-\rho T_\varphi)]+\varepsilon\delta^{-1/2}\,[\mu K\,(T_\eta\sigma^{-1}-2uu_\eta)]+$$
$$+O\,(\varepsilon\delta^{1/2})+O\,(\varepsilon^2\delta^{-1}) \tag{4.14}$$

5. The case $\lim\,\varepsilon\delta^{-1}=\beta_0<\infty$ for ε, $\delta\to 0$. Let us represent δ in the form of the product $\varepsilon\beta^{-1}$, where $\beta=O\,(1)$ when ε, $\delta\to 0$, and substitute $\delta=\varepsilon\beta^{-1}$ into Eqs. (4.2)–(4.14). As a result we obtain the equations of gas motion in the form

$$(ru\rho)_S+[r(1+KN\beta)\,\rho v^*]_N=O\,(\varepsilon) \tag{5.1}$$

$$\rho\left(\frac{uv_S{}^*\beta^2}{1+KN\beta}+v^*v_N{}^*\beta^2-\frac{Ku^2\beta}{1+KN\beta}\right)+p_N=O\,(\varepsilon) \tag{5.2}$$

$$\rho\left(\frac{uu_S}{1+KN\beta}+v^*u_N+\frac{Kuv^*\beta}{1+KN\beta}\right)+\frac{p_S}{1+KN\beta}=O\,(\varepsilon) \tag{5.3}$$

$$\frac{uw_S}{1+KN\beta}+v^*w_N=O\,(\varepsilon) \tag{5.4}$$

$$\rho\left(\frac{uT_S}{1+KN\beta}+v^*T_N\right)-\left(\frac{up_S}{1+KN\beta}+p_Nv^*\right)=O(\varepsilon)$$

$$(v=v^*\beta,\ N=n\varepsilon^{-1},\ S=s\beta\varepsilon^{-1}) \tag{5.5}$$

$$(r\rho u_S)+[r(1+K\eta\varepsilon^{1/2}\beta^{1/2}\rho v^\circ)]_\eta=O(\varepsilon) \tag{5.6}$$

$$p_\eta-K\rho u^2(\varepsilon\beta)^{1/2}=O(\varepsilon) \tag{5.7}$$

$$\rho\left(\frac{uu_S}{1+K\eta(\varepsilon\beta)^{1/2}}+v^\circ u_\eta+\frac{Kuv^\circ}{1+K\eta(\varepsilon\beta)^{1/2}}(\varepsilon\beta)^{1/2}\right)+\frac{p_S}{1+K\eta(\varepsilon\beta)^{1/2}}=$$

$$=(\mu u_\eta)_\eta+[2K\mu u_\eta-(Ku\mu)_\eta](\varepsilon\beta)^{1/2}+O(\varepsilon) \tag{5.8}$$

$$\rho\left(\frac{uw_S}{1+K\eta(\varepsilon\beta)^{1/2}}+v^\circ w_\eta\right)=(\mu w_\eta)_\eta+K\mu w_\eta(\varepsilon\beta)^{1/2}+O(\varepsilon) \tag{5.9}$$

$$\rho\left(\frac{uT_S}{1+K\eta(\varepsilon\beta)^{1/2}}+v^\circ T_\eta\right)-\left(\frac{up_S}{1+K\eta(\varepsilon\delta)^{1/2}}+v^\circ p_\eta\right)=\sigma^{-1}(\mu T_\eta)_\eta+$$

$$+\mu(u_\eta^2+w_\eta^2)+(\varepsilon\beta)^{1/2}[\mu K(T_\eta\sigma^{-1}-2uu_\eta)]+O(\varepsilon) \tag{5.10}$$

$$(v=(\varepsilon\beta)^{1/2}v^\circ,\ \eta=n\beta^{1/2}\varepsilon^{-1/2},\ S=s\beta\varepsilon^{-1})$$

All equations in Sects. 4 and 5 are obtained with uniform accuracy defined by the factor $1+O(\varepsilon)$.

6. It is noted that in Eqs. (5.1)–(5.10) the derivatives of the function in respect to φ sought for, are absent. This means that with the accuracy accepted in the theory of boundary layer, the gas flow parameters in the boundary layer in meridional planes ($\varphi=$ const) can be defined independently. (We consider, of course, the neighborhood of the line $s=0$ where the generatrix of the body of revolution is greatly curved). In Eqs. (4.3)–(4.14) the derivatives of the quantities sought for in respect to φ have the factor δ, therefore the relation between the gas flow parameters in meridional planes $\varphi=$ const is weak if δ is small. (This dependence, apparently, can be considered by means of iteration).

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